Composition of Dynamic Control Objectives Based on Differential Games

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- This design allows to easily handle multiple, possibly conflicting, dynamically changing objectives
- The simplicity is in the independent specification of each objective from which the final overall controller is formed

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2. Define a differential game where each player controls an input and tries to achieve the corresponding objective
3. Compute a Nash equilibrium and use it to construct a controller that balances between the objectives

Virtual Inputs

- We propose a method to transform a linear time-invariant (LTI) system of the form:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

- To an equivalent system:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \sum_{i=1}^{N} B_i \mathbf{v}_i(t)$$

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- Each $\mathbf{v}_i$ is called a virtual input designed s.t. each input affects mostly one of the objectives

- For nonlinear systems, we use on-the-fly linearization around each state using linear parameter varying (LPV) models

Cost Functions

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- Each virtual input is associated with objective defined as an independent cost function of the form:

$$J_i = \int_0^\infty \left[\mathbf{x}(t)^T Q_i \mathbf{x}(t) + \sum_{j=1}^{N} \mathbf{v}_j(t)^T R_{ij} \mathbf{v}_j(t) \right] dt$$

where each Q_i specifies the cost of deviations from the zero state and R_{ij} specifies control costs

Movie

Nash Equilibrium

- We consider the feedback control policies $(\mathbf{v}_1^*, \mathbf{v}_2^*, \ldots, \mathbf{v}_N^*)$ where $\mathbf{v}_i^*$ maps the state to the i'th virtual input
- Such a tuple constitutes a Nash Equilibrium iff for all $1 \leq i \leq N$ and any policy $\mathbf{v}_i$ we have:

$$J_i(\mathbf{v}_1^*, \ldots, \mathbf{v}_i^*, \ldots, \mathbf{v}_N^*) \leq J_i(\mathbf{v}_1^*, \ldots, \mathbf{v}_i, \ldots, \mathbf{v}_N^*)$$

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- We propose to construct the actual input using the virtual inputs satisfying the Nash equilibrium

- This generates a control strategy that forms a dynamic balance between the objectives

Calculating the Nash Equilibrium

- For the Nash Equilibrium, we consider linear feedback controllers of the form:[1]

$$\mathbf{v}_i^*(\mathbf{x}(t)) = -K_i\mathbf{x}(t) = -R_{ii}^{-1}B_i^T P_i\mathbf{x}(t)$$

[1] Vassilis L. Syrmos Frank L. Lewis Draguna L. Vrabie. "Optimal Control". In: chap. 10, pp. 438–460.

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- where $\{P_i\}_{i=1}^N$ is a solution of the Ricatti equations:

$$0 = P_i A_c + A_c^T P_i + Q_i + \sum_{j=1}^{N} P_j B_j R_{jj}^{-T} R_{ij} R_{jj}^{-1} B_j^T P_j$$

- and where:

$$A_c = A - \sum_{i=1}^{N} B_i R_{ii}^{-1} B_i^T P_i$$

[1] Frank L. Lewis, "Optimal Control".

Quadrotor Example Use Case

- For demonstration, we implemented a quadrotor controller

[2] Amir Shapiro Hanoch Efraim Shai Arogeti and Gera Weiss. "Vision Based Output Feedback Control of Micro Aerial Vehicles in Indoor Environments". In: (2017).

Quadrotor Example Use Case

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2. The second one is a dynamic objective on the forward velocity, where the objective changes with the distance of the quadrotor to the wall
    - If the quadrotor is closer to the center than the wall, then the objective is to reach a (positive) constant velocity
    - Otherwise, the objective is to decrease the velocity to zero

There might be a conflict between these two objectives, thus finding a balance between them is beneficial

Quadrotor Control

- Let us consider the state vector of the quadrotor:

$$\mathbf{x} = \begin{bmatrix} \phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi} \ z \ \dot{z} \ x \ \dot{x} \ y \ \dot{y} \end{bmatrix}^T$$

- The quadrotor is controlled by applying desired torques using its motors
- The torques are generated by controlling the angular velocity of each motor
- Let us define the following input vector for the thrust and three torques along the axes:

$$\mathbf{u} = \begin{bmatrix} T \ \tau_x \ \tau_y \ \tau_z \end{bmatrix}^T$$

Quadrotor Model

We consider the following rigid-body non-linear model[3] (assuming small angles, symmetry and diagonal inertia matrix):

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{\phi} \\ \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + \frac{J_r}{I_{xx}}\dot{\theta}\Omega_r + \frac{l}{I_{xx}}\tau_x \\ \dot{\theta} \\ \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\phi}\dot{\psi} - \frac{J_r}{I_{yy}}\dot{\phi}\Omega_r + \frac{l}{I_{yy}}\tau_y \\ \dot{\psi} \\ \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\theta}\dot{\phi} + \frac{1}{I_{zz}}\tau_z \\ \dot{z} \\ g - \frac{1}{m} cos\phi \cos\theta \; T \\ \dot{x} \\ \frac{1}{m}\left(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi\right) T \\ \dot{y} \\ \frac{1}{m}\left(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi\right) T \end{bmatrix}$$

[3] Samir Bouabdallah. "Design and Control of Quadrotors with Application to Autonomous Flying". In: (2007).

Simulation Tactics I

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- We show the performance of the suggested methodology in the context of the quadrotor hierarchical control strategy
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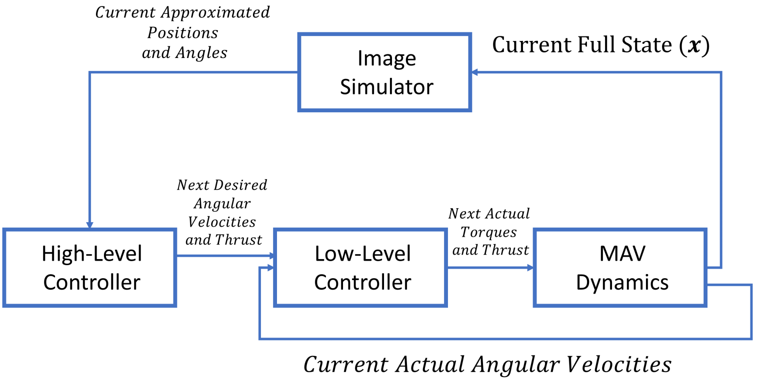
- We assume the state of the system is obtained by a linearization of the model proposed earlier
- Then we simulate the corresponding image that the quadrotor is presumably witnessing according to that state

Simulation Tactics II

The following is a high-level representation of the simulated system:



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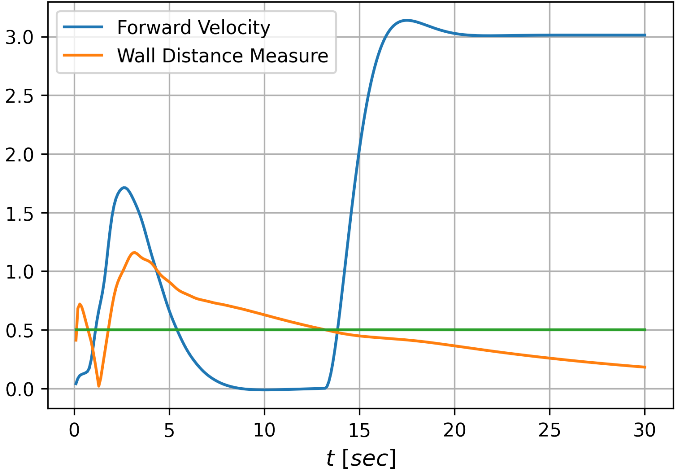
2. This state is fed into the image simulator that produces the current corresponding approximated positions and angles
3. Then the high-level controller produces the desired thrust and angular rates
4. Finally, the low-level controller produces the actual thrust and torques, $\mathbf{u}$, to be applied to the quadrotor

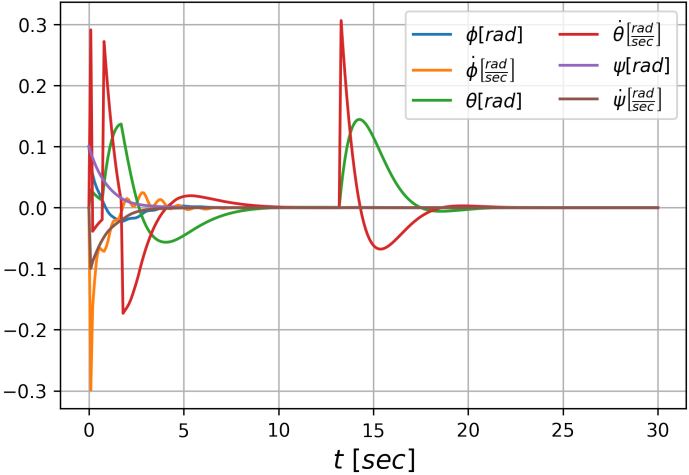
Results I - Objectives Dynamics

The following figure displays the simulation results. Once the wall measure reaches 0.5 - the velocity starts to increase from 0 to 3:



Results II - Angular State Variables

The following figure displays the angles and angular velocities



Conclusions

- We saw our method provides an easy way of finding a plausible balance in between possibly conflicting objectives
- Our simulation provides a quick stabilization of the state variables, even after changes in the interaction
- Handling non-linear systems with multiple objectives can be made quite easy by this method

Python Package

- We designed a Python package that encapsulates the method described
- The package provides a quick way to define objectives and generates the corresponding dynamic differential games
- Then the algorithm solves these games on-the-fly and generates the state variables
- The package is called `PyDiffGame` and is detailed extensively in the article

PyDiffGame

```
git clone https://github.com/krichelj/PyDiffGame.git
```

Thank you for listening!

Acknowledgments

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